Learning of Network Structure from Neuronal Spike Train Data

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Neural network in brain

Figure 1: *Illustration of neural network in brain*

Neural spike train data

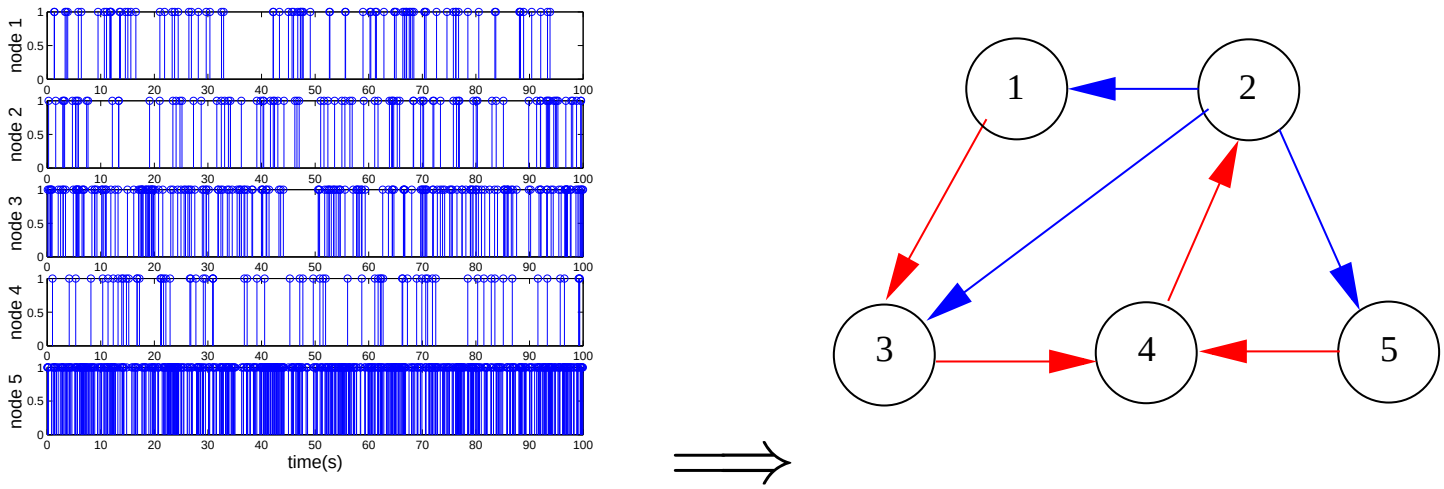

Figure 2: *Stem plot of neural spike train data (left) and underlying neural network (right).*

- **Goal**: to recover the underlying network structure from the observations of spike train data.

Model-based approach

- Node set (neural ensemble)

$$\mathcal{V} = \{1, 2, \ldots, V\}$$

- Point process (spike train data)

$$\boldsymbol{T}_i = (T_{i,1}, \ldots, T_{i,N_i}), \quad i \in \mathcal{V}$$

- Counting process (spike counts)

$$\mathrm{N}_i(t) \quad = \quad \sum_{\ell=1}^{N_i} \mathrm{I}(0 \leq T_{i,\ell} \leq t)$$

- Intensity process (firing rate)

$$\lambda_i(t) \quad = \quad \lim_{\Delta_t \downarrow 0} \frac{1}{\Delta_t} \mathrm{P}\{\mathrm{N}_i(t + \Delta_t) = \mathrm{N}_i(t) + 1 \mid \mathscr{F}_t\}$$

Modeling $\lambda_i(t)$

Proposed continuous-time GLM model

$$\lambda_i(t) \quad = \quad \exp\left\{\beta_{0;i} + \sum_{j \in \mathcal{V} \setminus i} \beta_{j,i}\, x_j(t)\right\}, \quad i \in \mathcal{V}, \quad t \in [0, T].$$

- $\beta_{j,i}$: connection strength parameter.

$$\begin{array}{lll} \beta_{j,i} > 0: & \text{excitatory} & \text{effect from node } j \text{ on node } i\ ; \\ \beta_{j,i} = 0: & \text{no} & \text{effect from node } j \text{ on node } i\ ; \\ \beta_{j,i} < 0: & \text{inhibitory} & \text{effect from node } j \text{ on node } i\ . \end{array}$$

Parameter estimation

$$\lambda_i(t) \quad = \quad \exp\left\{\beta_{0;i} + \sum_{j \in \mathcal{V} \setminus i} \beta_{j,i}\, x_j(t)\right\} := \exp\left\{\widetilde{\boldsymbol{\beta}}_i^{\mathsf{T}} \cdot \widetilde{\boldsymbol{x}}_i(t)\right\}.$$

Proposed penalized M-estimator $\widehat{\widetilde{\boldsymbol{\beta}}}_i$

$$\widehat{\widetilde{\boldsymbol{\beta}}}_i \quad = \quad \arg\min_{\widetilde{\boldsymbol{\beta}}_i \in \mathbb{R}^V} \left\{\mathcal{L}_i(\widetilde{\boldsymbol{\beta}}_i) + \mathcal{P}(\widetilde{\boldsymbol{\beta}}_i)\right\}.$$

- Loss function: negative log-likelihood function (Ozaki 1979)

$$\mathcal{L}_i(\widetilde{\boldsymbol{\beta}}_i) = -\frac{1}{T}\left[\int_0^T \log\{\lambda_i(t)\}\mathrm{dN}_i(t) - \lambda_i(t)\mathrm{d}t\right].$$

- Estimated network:

$$\text{Excitatory effects} \quad \widehat{\mathcal{E}}^{+} \quad = \quad \{(j,i) : \widehat{\beta}_{j,i} > 0\}$$

$$\text{Inhibitory effects} \quad \widehat{\mathcal{E}}^{-} \quad = \quad \{(j,i) : \widehat{\beta}_{j,i} < 0\}$$

Real data experiment

- Data: the prefrontal cortex spike train dataset “pfc-6” (of 55 neurons) on CRCNS.
- Estimation result:

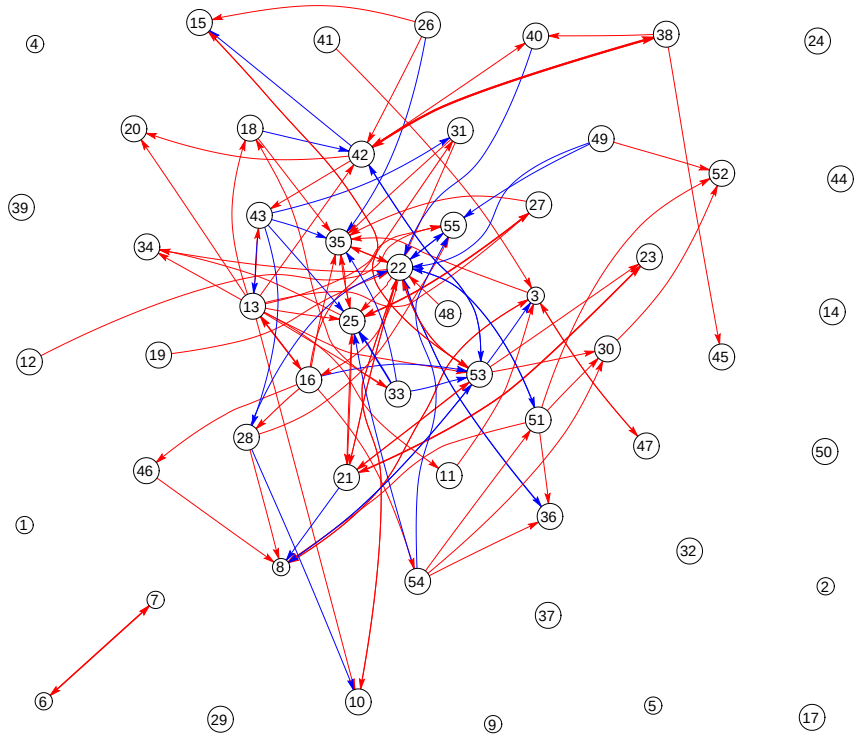

Figure 3: *Estimated network by our method. Red arrow is excitatory effect, Blue arrow is inhibitory effect.*